<table>
<thead>
<tr>
<th>Grade 10</th>
<th>Grade 11</th>
<th>Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Revise basic results established in earlier grades.</td>
<td>(a) Investigate and prove theorems of the geometry of circles assuming results from earlier grades, together with one other result concerning tangents and radii of circles.</td>
<td>(a) Revise earlier (Grade 9) work on the necessary and sufficient conditions for polygons to be similar.</td>
</tr>
<tr>
<td>(b) Investigate line segments joining the midpoints of two sides of a triangle.</td>
<td>(b) Solve circle geometry problems, providing reasons for statements when required.</td>
<td>(b) Prove (accepting results established in earlier grades):</td>
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<tr>
<td>(c) Properties of special quadrilaterals.</td>
<td>(c) Prove riders.</td>
<td>• that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Mid-point Theorem as a special case of this theorem);</td>
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</table>

### REVISION FROM EARLIER GRADES

#### SIMILARITY

<table>
<thead>
<tr>
<th>AAA or ∆∆∆</th>
<th>[ \triangle ABC \parallel \triangle DEF ]</th>
</tr>
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</table>

<table>
<thead>
<tr>
<th>SSS</th>
<th>[ \frac{MN}{RS} = \frac{ML}{RT} = \frac{NL}{ST} ]</th>
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</thead>
</table>

#### CONGRUENCY

<table>
<thead>
<tr>
<th>SSS</th>
<th>[ \triangle PQR = \triangle STU ]</th>
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<table>
<thead>
<tr>
<th>AAS</th>
<th>[ \triangle UVW = \triangle XYZ ]</th>
</tr>
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</table>

<table>
<thead>
<tr>
<th>SAS (included angle)</th>
<th>[ \triangle FGH = \triangle IJK ]</th>
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<table>
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<tr>
<th>RHS</th>
<th>[ \triangle ABC = \triangle DEF ]</th>
</tr>
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</table>
## Properties of Special Quadrilaterals

<table>
<thead>
<tr>
<th>Quadrilateral</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parallelogram</strong></td>
<td>• Both pairs of opposite sides are parallel&lt;br&gt;• Both pairs of opposite sides are equal&lt;br&gt;• Both pairs of opposite angles are equal&lt;br&gt;• Diagonals bisect each other</td>
</tr>
<tr>
<td><strong>Rectangle</strong></td>
<td>All properties of parallelogram PLUS:&lt;br&gt;• Both diagonals are equal in length&lt;br&gt;• All interior angles are equal to 90°</td>
</tr>
<tr>
<td><strong>Rhombus</strong></td>
<td>All properties of parallelogram PLUS:&lt;br&gt;• All sides are equal&lt;br&gt;• Diagonals bisect each other perpendicularly&lt;br&gt;• Diagonals bisect interior angles</td>
</tr>
<tr>
<td><strong>Square</strong></td>
<td>All properties of a rhombus PLUS:&lt;br&gt;• All interior angles are 90°&lt;br&gt;• Diagonals are equal in length</td>
</tr>
<tr>
<td><strong>Kite</strong></td>
<td>Two pairs of adjacent sides are equal&lt;br&gt;• Diagonal between equal sides bisects other diagonal&lt;br&gt;• One pair of opposite angles are equal (unequal sides)&lt;br&gt;• Diagonal between equal sides bisects interior angles (is axis of symmetry)&lt;br&gt;• Diagonals intersect perpendicularly</td>
</tr>
<tr>
<td><strong>Trapezium</strong></td>
<td>One pair of opposite sides are parallel</td>
</tr>
</tbody>
</table>

### How to Prove That a Quadrilateral Is a Parallelogram

Prove any ONE of the following:
- Prove that both pairs of opposite sides are parallel
- Prove that both pairs of opposite sides are equal
- Prove that both pairs of opposite angles are equal
- Prove that the diagonals bisect each other
- Prove that ONE pair of sides are equal and parallel
HOW TO PROVE THAT A PARALLELOGRAM IS A RHOMBUS

Prove ONE of the following:

- Prove that the diagonals bisect each other perpendicularly
- Prove that any two adjacent sides are equal in length

TRIANGLES BETWEEN PARALLEL LINES

The AREA of two triangles on the SAME (OR EQUAL) BASE between two parallel lines, are EQUAL.

\[
\text{Area of } \triangle ABC = \text{Area of } \triangle ABD
\]

MIDPOINT THEOREM

The line segment joining the midpoints of two sides of a triangle, is parallel to the third side of the triangle and half the length of that side.

( Midpt Theorem )

If \( AD = DB \) and \( AE = EC \), then \( DE \parallel BC \) and \( DE = \frac{1}{2}BC \)

CONVERSE OF MIDPOINT THEOREM

If a line is drawn from the midpoint of one side of a triangle parallel to another side, that line will bisect the third side and will be half the length of the side it is parallel to.

( line through midpoint \( \parallel \) to 2nd side )

If \( AD = DB \) and \( DE \parallel BC \), then \( AE = EC \) and \( DE = \frac{1}{2}BC \).
Note: THEOREMS OF WHICH PROOFS ARE EXAMINABLE ARE INDICATED WITH

Theorem 1
If AC = CB in circle O, then OC ⊥ AB.
(line from centre to midpt of chord)

Converse of Theorem 1
If OC ⊥ chord AB, then AC = BC.
(line from centre ⊥ to chord)

Theorem 2
The angle at the centre of a circle subtended by an arc/a chord is double the angle at the circumference subtended by the same arc/chord. \( \angle \text{at centre} = 2 \times \angle \text{at circumference} \)

Theorem 3
The angle on the circumference subtended by the diameter, is a right angle.
The angle in a semi-circle is 90°.
(\(\angle s \text{ in semi circle OR diameter subtends right angle}\))

Converse of Theorem 3
If \( \angle \) = 90°, then AB is the diameter of the circle.
(chord subtends 90° OR converse \(\angle s \text{ in semi circle}\))
Theorem 4
The angles on the circumference of a circle subtended by the same arc or chord, are equal.

Converse of Theorem 4
If a line segment subtends equal angles at two other points, then these four points lie on the circumference of a circle.

Corollary of Theorem 4
Equal chords subtend equal angles at the circumference of the circle.

Theorem 5
The opposite angles of a cyclic quadrilateral are supplementary.
\[ \hat{A} + \hat{C} = 180^\circ \]
\[ \hat{B} + \hat{D} = 180^\circ \]
(opp \( \angle \)s of cyclic quad)

Converse of Theorem 5
If the opposite angles of a quadrilateral are supplementary, then it is a cyclic quadrilateral.

(opp \( \angle \)s quad sup OR converse opp \( \angle \)s of cyclic quad)
Theorem 6
The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

(\text{ext } \angle \text{ of cyclic quad})

Converse of Theorem 6
If the exterior angle of a quadrilateral is equal to the opposite interior angle, then it is a cyclic quadrilateral.

(\text{ext } \angle = \text{int opp } \angle \text{ OR converse ext } \angle \text{ of cyclic quad})

Theorem 7
The tangent to a circle is perpendicular to the radius at the point of tangency.

(\text{tan } \perp \text{ radius OR tan } \perp \text{ diameter})

Converse of Theorem 7
If a line is drawn perpendicularly to the radius through the point where the radius meets the circle, then this line is a tangent to the circle.

(\text{line } \perp \text{ radius OR converse tan } \perp \text{ radius OR converse tan } \perp \text{ diameter})

Theorem 8
If two tangents are drawn from the same point outside a circle, then they are equal in length.

(\text{tans from common pt OR Tans from same pt})
Theorem 9 (Tan chord theorem)

The angle between the tangent to a circle and a chord drawn from the point of tangency, is equal to the angle in the opposite circle segment.

(\textit{tan chord theorem})

Converse of Theorem 9

If a line is drawn through the endpoint of a chord to form an angle which is equal to the angle in the opposite segment, then this line is a tangent.

(\textit{converse tan chord theorem OR } \angle \textit{ between line and chord})

THREE WAYS TO PROVE THAT A QUADRILATERAL IS A CYCLIC QUADRILATERAL

Prove that:

- one pair of opposite angles are supplementary
- the exterior angle is equal to the opposite interior angle
- two angles subtended by a line segment at two other vertices of the quadrilateral, are equal.
The Concept of Proportionality (Revision)

A \[\frac{6 \text{ cm}}{4 \text{ cm}} = 3 : 2\]  and  \[\frac{9 \text{ cm}}{6 \text{ cm}} = 3 : 2\]

Although,  \(AB : BC = DE : EF\) it does NOT mean that \(AB = DE, AC = DF\) or \(BC = EF\).

**Theorem 1**

A line drawn parallel to one side of a triangle that intersects the other two sides, will divide the other two sides proportionally.

\(\text{line} \parallel \text{one side of } \Delta\)

**Converse of Theorem 1**

If a line divides two sides of a triangle proportionally, then the line is parallel to the third side of the triangle.

\(\text{line divides two sides of } \Delta \text{ in prop}\)

If  \(\frac{AD}{DB} = \frac{AE}{EC}\) then  \(DE \parallel BC\).

**Theorem 2 (Midpoint Theorem)**

(Special case of Theorem 1)

The line segment joining the midpoints of two sides of a triangle, is parallel to the third side of the triangle and half the length of that side.

(midpt theorem)

If  \(AD = DB\) and  \(AE = EC\), then  \(DE \parallel BC\) and  \(DE = \frac{1}{2}BC\)

**Converse of Theorem 2**

If a line is drawn from the midpoint of one side of a triangle parallel to another side, that line will bisect the third side and will be half the length of the side it is parallel to.

(line through midpt \parallel to 2nd side)

If  \(AD = DB\) and  \(DE \parallel BC\), then  \(AE = EC\) and  \(DE = \frac{1}{2}BC\).
Theorem 3

The corresponding sides of two equiangular triangles are proportional and consequently the triangles are similar.

\[ \text{(\( \Delta \)s OR equiangular \( \Delta \)s)} \]

If \( \Delta ABC \parallel \Delta DEF \) then \( \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \)

Converse of Theorem 3

If the sides of two triangles are proportional, then the triangles are equiangular and consequently the triangles are similar.

\[ \text{(Sides of \( \Delta \) in prop)} \]

If \( \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \) then \( \Delta ABC \parallel \Delta DEF \)

Theorem 4

The perpendicular drawn from the vertex of the right angle of a right-angled triangle, divides the triangle in two triangles which are similar to each other and similar to the original triangle.

Corollaries of Theorem 4

\[ \Delta ABC \parallel \Delta DBA \]
\[ \therefore \frac{AB}{DB} = \frac{BC}{BA} = \frac{AC}{DA} \]
\[ \therefore AB^2 = BD \cdot BC \]

\[ \Delta ABC \parallel \Delta DAC \]
\[ \therefore \frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC} \]
\[ \therefore AC^2 = CD \cdot CB \]

\[ \Delta DBA \parallel \Delta DAC \]
\[ \therefore \frac{DB}{DA} = \frac{BA}{AC} = \frac{DA}{DC} \]
\[ \therefore AD^2 = BD \cdot DC \]

Theorem 5 (The Theorem of Pythagoras)

From the corollaries it can be proven that: \( BC^2 = AB^2 + AC^2 \)
TIPS TO SOLVING GEOMETRY RIDERS

- **READ-READ-READ** the information next to the diagram thoroughly
- **TRANSFER** all given information to the **DIAGRAM**
- Look for **KEYWORDS**, e.g.
  - TANGENT: What do the theorems say about tangents?
  - CYCLIC QUADRILATERAL: What are the properties of a cyclic quad?
- **NEVER ASSUME** something!
  - Don’t assume that a certain line is the DIAMETER of a circle unless it is clearly stated or unless you can prove it
  - Don’t assume that a point is the CENTRE of a circle unless it is clearly stated (“circle M” means “the circle with midpoint M”)
- **Set yourself “SECONDARY” GOALS**, e.g.
  - To prove that $AB = BC$ (primary goal), first prove that $\hat{A} = \hat{C}$ (secondary goal) and vice versa
  - To prove that line AC is a tangent (primary goal), first prove that the line is perpendicular to radius OB (secondary goal)
  - To prove that BC is the diameter of the circle (primary goal), first prove that $\hat{A} = 90^\circ$ (secondary goal)
- For questions like: Prove that $\hat{A}_1 = \hat{C}_2$. Start with **ONE PART**. Move to the **OTHER PART** step-by-step stating reasons. Remember it has to be clear and **logical** to the reader!
  - E.g. $\hat{A}_1 = \hat{A}_2 ; \hat{A}_2 = \hat{C}_1 ; \hat{C}_1 = \hat{C}_2 ; \therefore \hat{A}_1 = \hat{C}_2$
GRADE 11 GEOMETRY SAMPLE QUESTIONS

Question 1

AB and CD are two chords of the circle with centre O.

\[ OE \perp CD, AF = FB, OE = 4 \text{ cm}, OF = 3 \text{ cm} \text{ and } AB = 8 \text{ cm}. \]

Calculate the length of CD. \[8\]

Question 2

O is the centre of the circle. STU is a tangent at T.

\[ BC = CT, \quad \angle ATC = 105^\circ \text{ and } \angle CTU = 40^\circ \]

Calculate, giving reasons, the size of:

2.1 \( \hat{A}_2 \)  
2.2 \( \hat{A}_1 \)  
2.3 \( \hat{B} \)  
2.4 \( \hat{C}_2 \)  

\[13\]

Question 3

3.1 Write down with reasons four other angles which are equal to \( x \). \[8\]

3.2 Prove that \( \triangle ABC \parallel \triangle EDC \). \[4\]

3.3 Prove that \( AC = \frac{BC \cdot EC}{DC} \). \[2\]

\[14\]

Question 4

O is the centre of the circle. BC = CD

Express the following in terms of \( x \):

4.1 \( \hat{B}_2 \)  
4.2 \( BCD \)  
4.3 \( \hat{A} \)  

\[9\]
Question 5

LOM is the diameter of circle LMT. The centre of the circle is O. TN is a tangent at T.

\[ LN \perp NP \]

Prove that:

5.1 MNPT is a cyclic quadrilateral. \( \text{ (3) } \)
5.2 \( NP = NT \) \( \text{ (6) } \)

[9]

Question 6

PA and PC are tangents to the circle at C and A. AD \parallel PC and PD intersects the circle at B.

Prove that:

6.1 \( AC \) bisects \( P \hat{D} \) \( \text{ (6) } \)
6.2 \( \hat{B}_1 = \hat{B}_3 \) \( \text{ (6) } \)
6.3 \( A\hat{P}C = A\hat{B}D \) \( \text{ (4) } \)

[16]

Question 7

TA is a tangent to the circle. M is the centre of chord PT.

\[ TA \perp PA. \] O is the centre of the circle.

Prove that:

7.1 MTAR is a cyclic quadrilateral. \( \text{ (3) } \)
7.2 \( PR = RT \) \( \text{ (4) } \)
7.3 TR bisects PTA \( \text{ (4) } \)
7.4 \( \hat{T}_2 = \frac{1}{2} \hat{O}_1 \) \( \text{ (4) } \)

[15]
GRADE 12 GEOMETRY SAMPLE QUESTIONS

Example

Given: \( AD: DB = 2:3 \) and \( BE = \frac{4}{3} EC \).
Instruction: Determine the ratio of \( CP: PD \).
Solution:
In \( \Delta ABE \):
\[
\frac{BE}{KE} = \frac{5}{2} \quad \therefore \frac{BE}{KE} = \frac{5}{2} KE
\]
But it was given that \( BE = \frac{4}{3} EC \)
\[
\therefore \frac{4}{3} EC = \frac{5}{2} KE
\]
\[
\frac{EC}{KE} = \frac{5}{2} \div \frac{4}{3} = \frac{15}{8}
\]
In \( \Delta CDK \):
\[
\frac{CP}{PD} = \frac{CE}{EK} = \frac{15}{8}
\]
\( CP: PD = 15:8 \)

Question 1

\( ED = 22 \text{ cm}, DC = 33 \text{ cm}, BC = 15 \text{ cm} \) and \( AB = x \).
Calculate the value of \( x \). \[4\]

Question 2

\( BF || CE, BC = \frac{3}{8} AC \) and \( AE: ED = 4:3 \)
Determine the ratio \( DG: GB \). \[8\]

Question 3

\( \frac{RB}{RQ} = \frac{1}{3}, PA: AR = 1:2 \) and \( PM || AB \).

3.1 Write down the values of \( RA: RP \) and \( RB: BQ \). \[2\]

3.2 Determine \( BM: BR \) \[1\]

3.3 Prove that \( RM = MQ \). \[6\]
Question 4

Given: \( PQ \parallel BA \) and \( PR \parallel BD \)

Prove that \( QR \parallel AD \). \[4\]

Question 5

\( \Delta PQT \) is inscribed in a circle. \( AO \parallel QR, PA = AQ \) and \( PB = BT \)

PR is the diameter of the circle.

Prove that:

5.1 \( AB \parallel QT \) \hspace{1cm} (2)
5.2 \( O \) is the centre of the circle \hspace{1cm} (2)
5.3 \( BORT \) is a trapezium. \hspace{1cm} (2)

Question 6

Given: \( PA: AQ = 5:4 \) and \( PB: BR = 5:2 \)

\( S \) is the midpoint of \( AQ \)

6.1 Prove that \( AT = 2SR \) \hspace{1cm} (8)
6.2 If \( RK \parallel QX \), determine \( PX: XT \) \hspace{1cm} (6)

[14]

Question 7

Rectangle \( DEFK \) is drawn inside right-angled \( \Delta ABC \).

Prove that:

7.1 \( AD: BD = DE: BK \) \hspace{1cm} (4)
7.2 \( DE: EC = AE: FC \) \hspace{1cm} (4)
7.3 \( KF: EC = AE: FC \) \hspace{1cm} (1)
7.4 \( \frac{AB}{AD} = \frac{AC}{AE} \) \hspace{1cm} (3)

[12]
Question 8

ABOC is a kite with \( \hat{B} = \hat{C} = 90^\circ \)

8.1 Why is \( \triangle OCD \parallel \triangle OAC \)?

8.2 Complete:

8.2.1 \( OC^2 = \ldots \times \ldots \)

8.2.2 \( CA^2 = \ldots \times \ldots \)

8.2.3 \( CD^2 = \ldots \times \ldots \)

8.3 Prove that \( \frac{BB^2}{OB^2} = \frac{AD}{AO} \)

8.4 Prove that \( OC^2 - OD^2 = OD \cdot DA \)

8.5 If \( OD = \frac{1}{2} DA = x \), prove that \( CD = \sqrt{2} \cdot OD \)
MIXED EXERCISES

1. In the diagram, TBD is a tangent to circles BAPC and BNKM at B. AKC is a chord of the larger circle and is also a tangent to the smaller circle at K. Chords MN and BK intersect at F. PA is produced to D. BMC, BNA and BFKP are straight lines. Prove that:
   a) MN \parallel CA
   b) \( \triangle KMN \) is isosceles
   c) \( \frac{BK}{KP} = \frac{BM}{MC} \)
   d) DA is a tangent to the circle passing through points A, B and K.

2. In the diagram below, chord BA and tangent TC of circle ABC are produced to meet at R. BC is produced to P with RC=RP. AP is not a tangent. Prove that:
   a) ACPR is a cyclic quadrilateral.
   b) \( \triangle CBA \parallel \triangle RP A \)
   c) \( RC = \frac{CB \cdot RA}{AC} \)
   d) \( RB \cdot AC = RC \cdot CB \)
   e) Hence prove that \( RC^2 = RA \cdot RB \)

3. In the diagram alongside, circles ACBN and AMBD intersect at A and B. CB is a tangent to the larger circle at B. M is the centre of the smaller circle. CAD and BND are straight lines. Let \( A_3 = x \)
   a) Determine the size of \( \widehat{D} \) in terms of \( x \).
   b) Prove that:
      i) \( CB \parallel AN \)
      ii) \( AB \) is a tangent to circle ADN.
4. In the diagram below, O is the centre of circle ABCD. DC is extended to meet circle BODE at point E. OE cuts BC at F. Let $\theta_1 = x$.

a) Determine $\hat{A}$ in terms of $x$.

b) Prove that:
   i) $BE = EC$
   ii) $BE$ is NOT a tangent to circle ABCD.

5. In the diagram alongside, medians AM and CN of $\triangle ABC$ intersect at O. BO is produced to meet AC at P. MP and CN intersect in D. OR $\parallel$ MP with R on AC.

a) Calculate, giving reasons, the numerical value of $\frac{ND}{NC}$.

b) Use $AO: AM = 2:3$, to calculate the numerical value of $\frac{RP}{PC}$.

6. In the diagram, AD is the diameter of circle ABCD. AD is extended to meet tangent NCP in P. Straight line NB is extended to Q and intersect AC in M with Q on straight line ADP. AC $\perp$ NQ at M.

a) Prove that NQ $\parallel$ CD.

b) Prove that ANCQ is a cyclic quadrilateral.

c) i) Prove that $\triangle PCD \parallel \triangle PAC$.
   ii) Hence, complete: $PC^2 = \ldots$

d) Prove that $BC^2 = CD \cdot NB$

e) If it is further given that PC=MC, prove that

$$1 - \frac{BM^2}{BC^2} = \frac{AP \cdot DP}{CD \cdot NB}$$
1. a) \( \hat{B}_3 = \hat{M}_1 \) \( \text{tan chord} \)
\[ B_1 = \hat{C} \]
\[ \therefore \hat{M}_1 = \hat{C} \]
\[ \therefore MN \parallel CA \quad \text{corr } \angle s = \]
b) \( \hat{R}_3 = \hat{M}_2 \) \( \text{alt } \angle s \)
\[ R_1 = \hat{N}_2 \] \( \text{tan chord} \)
\[ \therefore \Delta KMN \text{ is isosceles} \]
c) \( \hat{R}_4 = \hat{N}_2 \) \( \text{alt } \angle s \)
\[ \hat{N}_2 = \hat{B}_3 \] \( \angle s \text{ in same segment} \)
\[ \hat{A}_3 = \hat{B}_3 \] \( \angle s \text{ in same segment} \)
\[ \therefore \hat{R}_4 = \hat{A}_3 \]
\[ \therefore NK \parallel AP \quad \text{alt } \angle s = \]
\[ \frac{BN}{NA} = \frac{BK}{KP} \] \( \text{line } \parallel \text{ to one side of } \Delta \)
But \( \frac{BN}{NA} = \frac{BM}{MC} \) \( \text{line } \parallel \text{ to one side of } \Delta \)
\[ \frac{BK}{KP} = \frac{BM}{MC} \]
d) \( \hat{A}_3 = \hat{B}_3 \) \( \angle s \text{ in same segment} \)
\( \hat{B}_3 = \hat{B}_2 \) \( \text{equal chords subt equal } \angle s \)
\[ \therefore \hat{A}_3 = \hat{B}_2 \]
\[ \therefore DA \text{ is a tangent to the circle through } A, B \text{ and } K \]

2. a) \( \hat{C}_3 = C\hat{P}R \) \( \angle s \text{ opp equal sides} \)
\[ \hat{C}_3 + \hat{C}_2 = \hat{A}_1 + \hat{B} \quad \text{ext } \angle \text{ of } \Delta \]
\[ \hat{C}_2 = \hat{B} \] \( \text{tan chord} \)
\[ \therefore \hat{C}_3 = \hat{A}_1 \]
\[ \therefore \hat{A}_1 = C\hat{P}R \text{ both } = \hat{C}_3 \]
ACPR is a cyclic quadrilateral \( (\text{ext } \angle \text{ of quad}) \)
b) In \( \Delta CBA \) and \( \Delta RPA \):
\[ \hat{P}_2 = \hat{C}_2 \] \( \angle s \text{ in same segment} \)
\[ = \hat{B} \] \( \text{proven in 2 a} \)
\[ \therefore \hat{B} = \hat{P}_2 \]
\[ \hat{C}_1 = A\hat{R}P \] \( \text{ext } \angle \text{ of cyclic quad} \)
\[ \hat{A}_1 = \hat{A}_3 \] \( 3^{rd} \angle \text{ of } \Delta \)
\[ \therefore \Delta CBA \parallel \Delta RPA \quad \angle \angle \angle \]
c) \( \frac{RP}{CB} = \frac{RA}{CA} \) \( \text{from 2 b} \)
\[ RP = \frac{CB.RA}{CA} \text{ but } RP = RC \]
\[ \therefore RC = \frac{CB.RA}{CA} \]
d) In \( \Delta RAC \) and \( \Delta RCB \):
\[ \hat{C}_2 = \hat{B} \] \( \text{tan chord} \)
\[ \hat{R}_1 \text{ is common} \]
\[ R\hat{C}B = R\hat{A}C \] \( 3^{rd} \text{ angle} \)
\[
\therefore \triangle BAC \parallel \triangle BCB \quad \angle \angle \\
\frac{AC}{CB} = \frac{RC}{RB} \quad \Delta s \ |||
\]
\[
RB \cdot AC = RC \cdot CB
\]
e) \[
\frac{CB}{RP} = \frac{CA}{RA} \quad \text{from 2. b)}
\]
\[
\frac{CB}{CR} = \frac{CA}{RA} \quad RC = RP
\]
\[
AC = \frac{CR \cdot RA}{RC}
\]
From 2.d) \[
AC = \frac{RC \cdot CB}{RB}
\]
\[
\therefore \frac{CB \cdot RA}{RC} = \frac{RC \cdot CB}{RB}
\]
\[
\therefore RC^2 = RA \cdot RB
\]

3. a) \[
\hat{B}_2 = \hat{A}_3 = x \quad \angle s \text{ opp equal sides}
\]
\[
\hat{M}_1 = 180^\circ - 2x \quad \text{sum } \angle s \text{ of } \Delta
\]
\[
\therefore \hat{D} = 2x
\]
b. i) \[
\hat{C} = \frac{\hat{M}_1}{2} \quad \angle \text{ at centre } = 2x \angle \text{ circ}
\]
\[
= 90^\circ - x
\]
\[
\triangle C \hat{B}D = 180^\circ - (90^\circ - x + 2x) \quad \text{sum } \angle s \text{ of } \Delta
\]
\[
= 90^\circ - x
\]
\[
\hat{N}_1 = \hat{C} = 90^\circ - x \quad \text{ext } \angle \text{ of cyclic quad}
\]
\[
\therefore \triangle C \hat{B}D = \hat{N}_1
\]
\[
\therefore \triangle C \parallel \triangle AN \quad \text{corr } \angle s
\]
b. ii) \[
\triangle C \hat{B}A = \hat{D} = 2x \quad \text{tang chord}
\]
\[
\triangle C \hat{B}A = \hat{A}_2 \quad \text{alt } \angle s
\]
\[
\hat{A}_2 = \hat{D}
\]
\[
\therefore \hat{A}B \text{ is a tangent } \angle \text{ betw line & chord}
\]

4. a) \[
\hat{B}_3 = \hat{E}_1 = x \quad \angle s \text{ in same segment}
\]
\[
\hat{B}_3 = \hat{D}_2 = x \quad \angle s \text{ opp } = \text{ sides}
\]
\[
\triangle B \hat{D}D = 180^\circ - 2x \quad \text{sum } \angle s \text{ of } \Delta
\]
\[
\hat{A} = 90^\circ - x \quad \angle \text{ at centre } = 2x \angle \text{ at circumference}
\]
b. i) \[
\hat{C}_1 = 90^\circ - x \quad \text{ext } \angle \text{ of cyclic quad}
\]
\[
\hat{F}_2 = 180^\circ - (x + 90^\circ - x) \quad \text{sum } \angle s \text{ of } \Delta
\]
\[
= 90^\circ
\]
In \triangle BFE and \triangle CEF:
\[
\hat{F}_1 = \hat{F}_2 = 90^\circ \quad \angle s \text{ on str line}
\]
BF = FC
FE is common
\[
\triangle BFE \equiv \triangle CEF \quad \text{s } \angle s
\]
BE = EC \quad \Delta s \ \equiv
b. ii) \[
\hat{B}_1 = 90^\circ - x \quad \text{sum } \angle s \text{ of } \Delta
\]
\[
\therefore \hat{B}_1 = \hat{A}
\]
\[
\therefore \hat{B}_1 \text{ is not a tangent } (\hat{B}_1 + \hat{B}_2 \neq \hat{A})
\]
5. a) \( P \) is midpoint of \( AC \)  
medians concur

\[ AB \parallel PM \]  
midpt theorem

In \( \triangle BNC \):
\[
\frac{ND}{NC} = \frac{BM}{BC} = \frac{AP}{AC} \\
= \frac{BM}{\frac{1}{2}BC} = \frac{1}{2}
\]

b) In \( \triangle AMP \):
\[
\frac{AO}{OM} = \frac{2OM}{OM} \\
\frac{RP}{AP} = \frac{RP}{AP} \\
\frac{PC}{AM} = \frac{AP}{OM} \\
= \frac{AM}{3OM} = \frac{1}{3}
\]

BP is a median

line \( \parallel \) one side of \( \triangle \)

6. a) \( \hat{C}_2 = 90^\circ \)  
\( \angle \) in semi \( \odot \)  

\( \hat{M}_2 = 90^\circ \)  
AM \bot NM

\( \therefore \ NQ \parallel CD \)  
corr \( \angle \)s=

b) \( \hat{C}_1 = \hat{N} \)  
\( || \) lines, corr \( \angle \)s

\( \hat{A}_2 = \hat{C}_1 \)  
tan chord

\( = \hat{N} \)

\( \therefore \ \text{ANCQ is a cyclic quad} \)  
\( \angle \)s subt by same line segm

c) i) In \( \triangle PCD \) and \( \triangle PAC \):

\( \hat{C}_1 = \hat{A}_2 \)  
tan chord

\( \hat{P} \) is common

\( \hat{D}_1 = A\hat{C}P \)  
3rd \( \angle \)

\( \therefore \ \angle PCD \parallel \angle PAC \)  
\( \angle \angle \)

c) ii) \( PC^2 = AP \cdot DP \)

d) In \( \triangle NBC \) and \( \triangle BCD \):

\( \hat{N} = \hat{A}_2 \)  
\( \angle \)s in same segm

\( = \hat{B}_2 \)  
\( \angle \)s in same segm

\( \hat{C}_4 = \hat{A}_1 \)  
tan chord

\( = \hat{D}_2 \)  
\( \angle \)s in same segm

\( \hat{B}_1 = B\hat{C}D \)  
3rd \( \angle \)

\( \therefore \ \angle NBC \equiv \angle BCD \)  
\( \angle \angle \)

\( \therefore \ \frac{BC}{NB} = \frac{CD}{NB} \)

\( BC^2 = CD \cdot NB \)

e) \[
1 - \frac{BM^2}{BC^2} = \frac{BC^2 - BM^2}{BC^2} \\
= \frac{MC^2}{BC^2} \\
= \frac{PC^2}{BC^2} \\
= \frac{AP \cdot DP}{CD \cdot NB} \quad \text{Pyth.}
\]